

**THE UNIVERSITY OF THE WEST INDIES**

**Mona Campus**

Semester l Semester II □ Supplemental/Summer School □

**Mid-Semester Examinations of: October /February/March** □ **/June** □ **2013/2014**

Course Code and Title: **COMP2201 Discrete Mathematics for Computer Scientists**

Date: **Friday, October 25, 2013** Time: **2:00 p.m.**

Duration: **1 Hour.** Paper No: **1 (of 1)**

Materials required:

**Answer booklet: Normal Special** □ **Not required** □

**Calculator: Programmable** □ **Non Programmable Not required** □

*(where applicable*)

**Multiple Choice answer sheets: numerical □ alphabetical □ 1-20 □ 1-100** □

Auxiliary/Other material(s) – Please specify: None

**Candidates are permitted to bring the following items to their desks: Pencil or pen, Ruler, ID card, Exam card**

**Instructions to Candidates: This paper has 2 pages & 6 questions.**

**Candidates are reminded that the examiners shall take into account the proper use of the English Language in determining the mark for each response.**

**All questions are COMPULSORY.**

**Calculators are allowed.**

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1. (a) Write the formula to find the number of integer solutions of

a1 + a2 + a3 + a4 = 20

subject to a1 ≥ 0, a2 > 1, a3 > 2, a4 ≥ 5 **[1]**

**10+4-1C4-1**

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| (b) | In an arithmetic series, the sum of the third term and the sixth term is  45. Three consecutive terms of the same series are 4*x* + 27, 2*x* + 26 |  |
|  | and 3*x* + 34. If the sum of the terms in the series is 105 |
|  | i. Find *x* | **[1]** |
|  | ii. Find the common difference, d | **[1]** |
|  | iii. Find the first term, a | **[1]** |
|  | iv. Find the number of terms in the series, n | **[2]** |
|  |  |  |

For AP, the difference between consecutive terms is the common difference, d

Therefore

d = (2x + 26) – (4x + 27)

= -2x - 1

Also

d = (3x + 34) – (2x + 26)

= x + 8

Equating

d = -2x - 1 = x + 8

⇒ -3x = 9

x = -9/3

x = -3

Therefore x is equal to -3

Using x as -7/3 to solve for d we have,

d = -2x - 1

= -2(-3) - 1

= 5

Therefore the common difference d, between the terms 5

u3 = a + 2d un = a + (n-1)d

u6 = a + 5d

u3 + u6 = 45

⇒  a + 2d + a + 5d = 45

⇒ 2a + 7d = 45

⇒ 2a + 7(5) = 45

⇒ 2a = (45 – 7(5))

⇒ 2a = (45 – 35)

⇒ a = 10/2

⇒ a = 5

Therefore the first term a, is 5

Sn = (n/2)(2a + (n-1)d)

105 = (n/2)(2(5) + (n-1)(5))

105 = (n/2)(10 + 5n - 5)

210 = n(5 + 5n)

210 = 5n + 5n^2

2100 = 50n + 50n^2

50n + 50n^2 – 2100 = 0

Factoring Quadratic Equation

50n + 50n^2 – 2100 = 0

50(n + n^2 - 42) = 0

50((n^2 – 6n ) + (7n - 42)) = 0

50(n(n – 6 ) + 7(n - 6)) = 0

50(n + 7)(n - 6) = 0

Either (n + 7) = 0 or (n - 6) = 0

n + 7 = 0

n = -7

Or

n – 6 = 0

n = 6

Since n has to be in the set of natural numbers then n must be equal to 6

Therefore the number of terms in the series n, is 6

1. Consider the recurrence function

*T(n) = 8T(n/2) + log n*

Give an expression for the runtime *T(n)* if the recurrence can be solved

with the Master Theorem. Assume that *T(n) = 1* for *n ≤ 1*. **[5]**

1. Let *f1*(*x*) and *f2*(*x*) be functions defined *fi* : *Z+ → R*

where *+* is the set of Positive integers and *R* is the set of Real numbers Prove the following statement

*If f*1 (*x*) (*g*1 (*x*)) *and*

*f*2 (*x*) (*g*2 (*x*)), then ( *f*1 *f*2 )(*x*) ((*g*1 *g*2 )(*x*))

Given f1(x) = Θ(g1(x)) and f2(x) = Θ(g1(x))

Show that f1f2(x) = Θ(g1g2(x))

f1(x) = Θ(g1(x))

⇒ C1 |g1(x)| ≤ f1(x) ≤ C2 |g1(x)|

Where C1 and C2 are constants

f2(x) = Θ(g2(x))

⇒ C3 |g2(x)| ≤ f2(x) ≤ C4 |g2(x)|

Where C3 and C4 are constants

**[4]**

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1. (a) Use the Binomial Theorem to show that **[3]**

*n*

2*n* *k C*(*n*, *k* ) 3*n*

*k* 0

We know that

We eliminate bk and substitute values for the proof

Let a = 2, b = 1

So we have,

.

.

As 1x = 1 for x

Therefore,

.

(b) What is the row of Pascal’s triangle containing the binomial coefficients **[1]**

5 

*k* , 0 *k* 5

.

Using Pascal’s Identity we can obtain the row

C(5, 0), C(5, 1), C(5, 2), C(5, 3), C(5, 4), C(5, 5)

= 1, 5, 10, 10, 5, 1

Therefore the row of Pascal’s triangle is 1, 5, 10, 10, 5, 1

|  |  |  |  |
| --- | --- | --- | --- |
| 5. | (a) | Cards of different colours and varied weights are placed in a bag. The Red card is three times as likely to be pulled as the Orange card and Yellow card. The Yellow card is two times as likely to be pulled as the Green and Blue cards. The Blue card is two times as likely to be pulled as the Indigo and Violet cards.  Assign probabilities to the seven outcomes in the sample space. | **[5]** |
|  | (b) | In a given town only 6 percent of all cases of domestic violence will be reported to the police. Find the probability that among 90 such cases in that town, at least two will be reported to the police. | **[3]** |

Using Binomial

. for x = 0,1,2,…,n

.

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|  |  |  |
| --- | --- | --- |
| 6. | Solve the following question using Tree Diagrams. A four-person committee |  |
|  | composed of Paul, Que, Rosie and Steve is to select a president, |  |
|  | vice president, and secretary. How many selections are there in which |  |
|  | Rosie is vice president? | **[3]** |
|  |  |  |

**END OF QUESTION PAPER**

**Mid-Semester I 2013/2014**